Investigation of rational airframe structural parameters

V. V. Chekhov
Central Aerohydrodynamic Institute (TsAGI), Moscow, Russia

1. Abstract

Rational designing of the aircraft frame on the base of optimality criteria method with consideration of material nonlinearity is presented in this contribution. In addition, influence of material nonlinearity on features of fully stressed design is considered with taking ultimate and limit loading systems into account.

2. Keywords

Application, Design, Structural, Aircraft, Nonlinear.

3. Introduction

Nonlinear effects must be taken into account in designing of units, to which high level loading applied. Real aircraft aggregates are structures in which many different materials are used. Amount of papers dealing with optimal design of structures made of several materials with taking nonlinearities into account is little. However nonlinear designing of such structures displays features which are absent in one-material structures designing. This paper describes rational designing of the maneuverable aircraft frame made with two materials considering physical nonlinearity.

4. Design model

The heavy frame of hypothetical maneuverable aircraft is considered. The design diagram of this structure is made up of finite elements (FE) of two kinds: frame caps are modeled by the rod elements (design variables are cross-sectional areas) and the web — by the membrane elements (design variables are thicknesses). The half of structure is considered in view of symmetry. The FE-model and discretized loading system are shown on Fig. 1. The certain case of the static ultimate loading [1] is a calculated case for the structure Because of the absence of buckling effects, breaking stresses (σ_b) are taken as the limit allowable stress levels. The limit (operational) loading [1] is taken into account too. Both the distribution of limit loads and the one of ultimate loads correspond to Fig. 1, the used safety factor is equal to 1.5. The limit loading must cause no residual strains in a structure, so engineering yield stresses ($\sigma_{0.2}$) are taken here as a limit allowable stress levels. Nonlinear behavior of materials under loading is described by the theory of small elasto-plastic deformations [2]. Used von Mises stress–strain diagrams are shown on the Fig. 2.

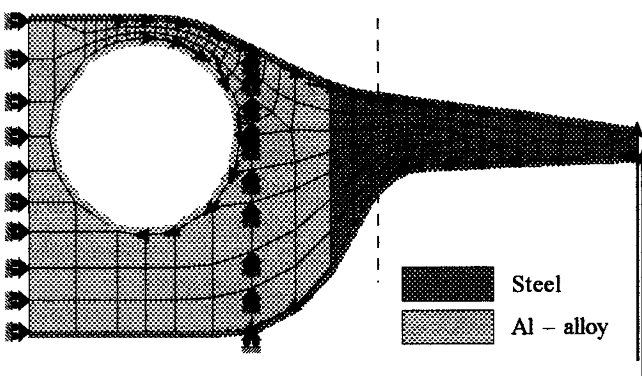

Figure 1.
FE-model of airframe.

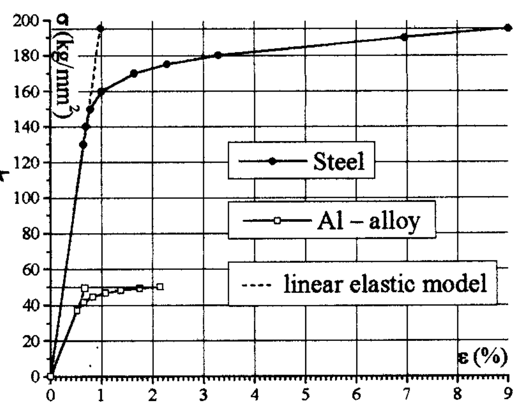

Figure 2.
Stress-strain diagrams of used materials.

This investigation considers the rational designing of the aircraft frame with use of the next optimality conditions for physically nonlinear structures which are made out of several materials [3]:

$$\begin{cases} \dfrac{\Pi_i}{\rho_i} = K = \text{const} & \text{if } \alpha_i > \overline{\alpha}_i \\ \dfrac{\Pi_i}{\rho_i} \le K & \text{if } \alpha_i = \overline{\alpha}_i \end{cases} \tag{1}$$

where i=1,...,n is number of current FE, n is amount of elements in the model, Π_i is average specific strain potential energy (on i-th FE volume), ρ_i is specific weight of the material of i-th FE, $\overline{\alpha}_i$ is minimal allowable value of the design variable α_i of the i-th FE. The conditions (1) are valid for the problem of structural mass minimization with prescribed structural compliance and side constraints and for the problem ofstructural compliance minimization with prescribed structural mass and side constraints. The next iterative algorithm for minimization of integral compliance of a structure with fixed weight is based on conditions (1):

$$\alpha_i^{(k+1)} = \alpha_i^{(k)} \frac{\left(\frac{\Pi_i^{(k)}}{\rho_i}\right)^{\frac{1}{q_i+1}}}{\Psi^{(k)}} , \qquad (2)$$

where i=1,...,n, k=1,2,..., parenthetic superscript denotes an iteration number, q_i is the exponent in power-type approximation of von Mises stress-strain diagram for the i-th FE material,

$$\Psi^{(k)} = \frac{\sum_{j=1}^{n} W_j^{(k)} \left(\frac{\Pi_j^{(k)}}{\rho_j}\right)^{\frac{1}{q_j+1}}}{W^{(0)}} , \qquad (3)$$

$W_j^{(k)}$ is the mass of j-th FE after k-th iteration of algorithm, $W^{(0)}$ is the prescribed mass of the structure. The optimality conditions (1) as well as the algorithm (2) have been described in detail in the contribution [3].

The fully stressed design (FSD) [4] is considered here as the initial approximation of the rational design. To build FSD the well-known heuristic algorithm (the stress ratio formula) [4] is used:

$$\alpha_i^{(k+1)} = \alpha_i^{(k)} \frac{\sigma_i^{(k)}}{\overline{\sigma}_i}, \qquad (4)$$

where σ_i is average von Mises stress, $\overline{\sigma}_i$ is maximal allowable stress level ($(\sigma_b)_i$ or $(\sigma_{0.2})_i$ depending on loading considered) of the i-th FE.

Some characteristic values of the frame designs developed below with algorithms (2) and (4) are compared in the Table 1.

5. Material nonlinearity effect in fully stressed design

At first, the design which is fully stressed under ultimate loading with allowable stress levels s_b was built on the base of the linear theory of elasticity (with use of the stress-strain diagrams shown on Fig. 2 by dashed lines). The distribution of dimensionless design variables in this FSD is shown on Fig. 3 where grayscale density of each FE is proportional to its design variable. It is usually expected that structures designed on the base of the linear elasticity theory have some reserve of strength which can be useful for further updating of an aircraft. However the analysis of the stress state made on the base of real stress-strain diagrams (Fig. 2) displays that the linear elastic FSD has no safety margin. Moreover, this nonlinear analysis detects that σ_b levels are exceeded in a significant part of structure. This can be seen from the Fig 4. Thus a linear FSD can not always be an overdesigned one. Therefore the analogous

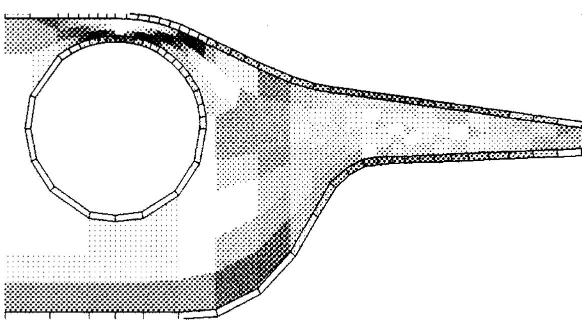

Fig 3.
Material distribution in FSD (ultimate loading) developed on the base of linear elasticity.

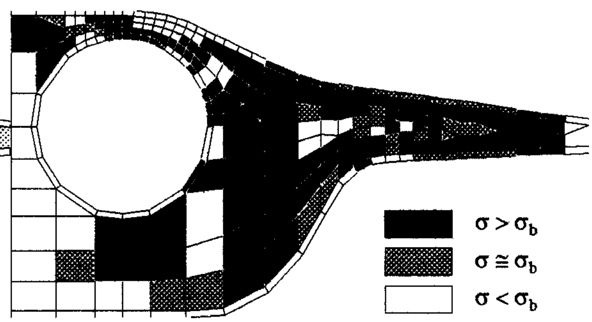

Fig 4.
Stress values in linear-elastic FSD (ultimate loading, see Fig 3), evaluated on the base of real behavior of materials.

physically nonlinear FSD was made. Its structural parameters distribution is shown on Fig. 5. Mass value of this design turned out to be more than one in the design shown on Fig. 3. In spite of the fact that the nonlinear FSD contains no elements with exceeded allowable stress level under ultimate loading (that can be seen in Fig. 6), stress analysis of this design under limit loading detects the exceeding of the limit stress levels ($\sigma_{0.2}$) in several elements. Fig. 7 demonstrates this. On the other hand the FSD built under the limit loading with an allowable stresses $\sigma_{0.2}$ (this FSD turned out to have material distribution similar to one shown on Fig. 3 but its mass value is less than the mass of the design shown on Fig. 3) is quite unallowable under the ultimate loading. Only simultaneous consideration of the ultimate loading and the limit loading allowed us to develop the FSD, which was acceptable under both considered systems of loading. This was made by means of variant of stress ratio algorithm (4):

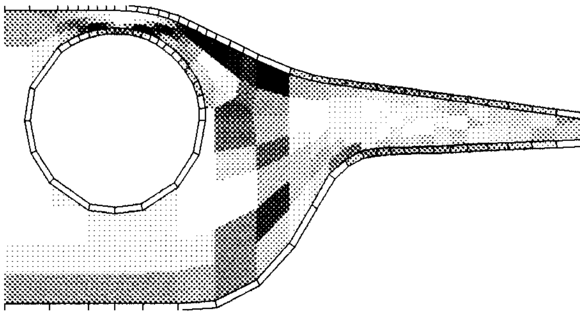

Fig 5.
Material distribution in FSD (ultimate loading) developed on the base of real behavior of materials.

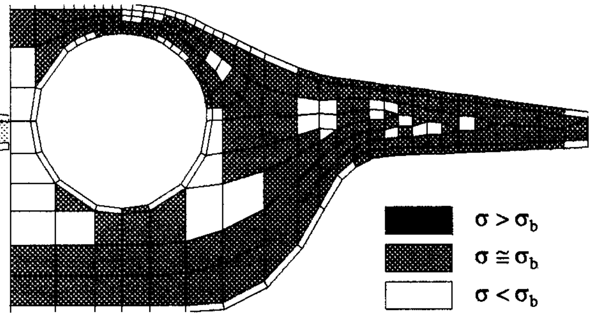

Fig 6.
Stress values in elasto-plastic FSD (ultimate loading, see Fig 5), evaluated on the base of real behavior of materials.

$$\alpha_i^{(k+1)} = \alpha_i^{(k)} \max\left(\frac{^{\text{ult}}\sigma_i^{(k)}}{(\sigma_b)_i}, \frac{^{\text{lim}}\sigma_i^{(k)}}{(\sigma_{0.2})_i} \right), \qquad (5)$$

where left superscript denotes appropriate loadcase. The resulting design is weakly differ from one shown on Fig. 5. This design from engineering point of view has not quite rational distribution of the structural parameters. This can be seen from Fig. 5: upper and lower steel booms around aluminum part of structure are not included into internal forces transfer that is disadvantageous for overall bending reception.

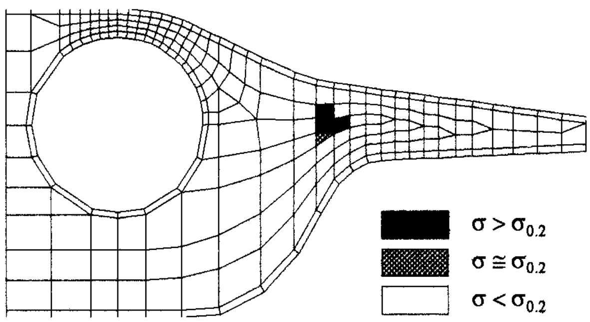

Fig 7.
Stress values in elasto-plastic FSD (ultimate loading, see Fig 5), evaluated under limit loading.

6. Minimization of compliance

The last design was subjected to compliance minimization algorithm (2) under ultimate loading. Use of this algorithm enabled us to decrease the maximal displacements at the structure more than on 73% (see Table 1). The distribution of the structural parameters (shown on Fig. 8) became rational and stress levels became less than their limit values both under ultimate loading and under limit loading. This design under ultimate loading meets the optimality condition (1).

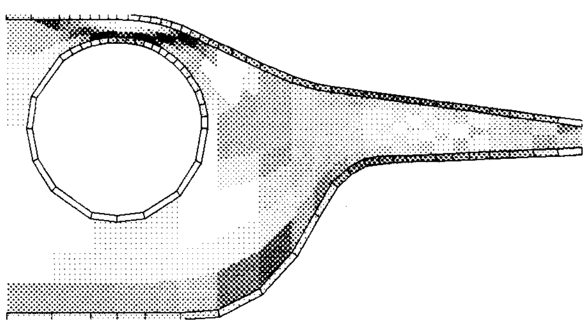

Fig 8.
Material distribution in structure after minimization of compliance of FSD.

7. Weight properties development

The last version of the frame (see Fig. 8) has some safety margin which is expected to allow to diminish mass on account of some increase of the structure compliance. For checking this opportunity the algorithm (4) (for ultimate loading) with fixation of σ_b value as the limit allowable stress level for the aluminum elements only and the smaller than σ_b value for the steel elements (this value was defined out of the optimality condition (1) with the maximum permissible value of K) was used. As a result the structural mass was decreased by 2,4% and the maximal displacement has some increase (see Table 1). The material distribution in this design is shown on the Fig. 9. The comparison with Fig. 8 demonstrates that this design is more rationale than design shown on Fig. 8: in the bottom part of structure the structural material has been concentrated in the steel chord. The design shown on Fig. 9 has got some margin of safety under limit loading.

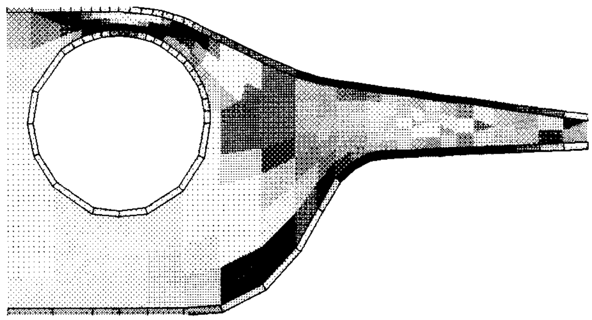

Fig 9.
Material distribution in design having maximal possible value of Π/ρ constant.

The studied structure can be considered as the complex of two parts weakly influencing each other by their structural parameters distributions (due to Saint Venant's principle). The boundary between them is approximately shown on the Fig. 1 by a dashed line. One of these parts consists of the steel elements only. Applying of optimality conditions (1) to each part individually shows that it is not necessary to set a value smaller than σ_b as allowable stress level for the beam-like part of structure containing one material. Therefore using of algorithm (4) was repeated with adequately assigned $\overline{\sigma}_i$ values (σ_b for aluminum elements and elements belonging to steel beam-like part of structure, and the values less than σ_b for the rest of elements). The resulted design is shown on Fig. 10. In spite of some "leak" of structural material from the steel bottom chord, the beam-like part of structure contains amount of material less than one on the Fig. 9 under the same total load. As a result the frame mass has been decreased by 6% (versus 2.4% in the design shown on Fig. 9, see Table 1).

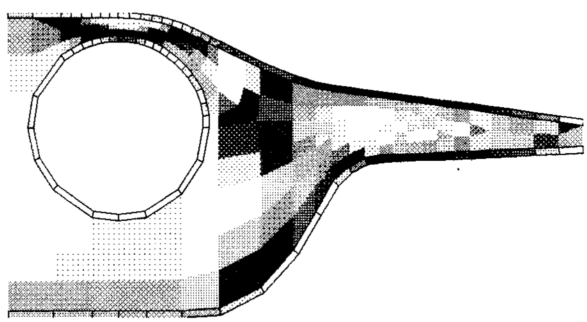

Fig 10.
Material distribution in design having maximal possible values of Π/ρ constant in each of 2 parts of structure.

8. Conclusions

The FSD designed with use of the linear theory of elasticity is not for each structure allowable with taking real nonlinear behavior of its materials into account under the same load.

Rational designing with taking material nonlinearity under ultimate loading into account does not warrant absence of residual strains under limit loading and therefore physically nonlinear structure acceptable under ultimate load requires the check of permissibility of the design under limit load.

Using of optimality conditions (1) allows to improve structural parameters of ineffectual FSD.

Design	Mass of structure	Under ultimate loads		Under limit loads	
		Total complementary energy	Maximal displacement	Total complementary energy	Maximal displacement
FSD (Ultimate loads, linear elastic) (Fig. 3)	92.6	—	—	100.3	99.5
FSD (Ultimate loads) (Fig. 5)	100.1	100.1	100.0	—	—
FSD (Limit loads)	74.7	—	—	124.2	123.2
FSD (Ultimate and limit loads)	100	100	100	100	100
Design with minimal compliance (Fig. 8)	100.2	56.2	26.6	90.3	88.8
Design with maximal Π/ρ (Fig. 9)	97.8	58.9	31.2	92.4	91.0
Design with maximal Π/ρ for each independent part of structure (Fig. 10)	94.0	91.3	86.6	98.5	97.5

Table 1. Some characteristic values of airframe design variants (in %).

9. References

[1] (1994). Aviation regulations. Part 25. Airworthiness standard. Transport category airplanes. Interstate Aviation Committee, 322 pp.

[2] Kachanov, L.M. (1969). Foundations of the theory of plasticity. Moscow, Nauka Publ., 420 pp.

[3] Selyugin, S.V. and Chekhov, V.V. (1999). Optimal physically nonlinear structures made of several materials. In: present collection.

[4] Melosh R.J. (1970). Convergence in fully-stressed designing. In: AGARD C.P. 36-70, pp. 7-1 – 7-17.